**3.2.7.6) Determine the weaker statement:**

**0 <= x <= 10 and 1<= x < 5.**

1 <= x < 5 ⇒ 0 <= x <= 10

<algebra> 1 <= x < 5 ⇒ (0 <= x < 1) ∨ (1 <= x < 5) ∨ (5 <= x <= 10)

<weakening/strengthening> T

Hence 0 <= x <= 10 is weaker.

**(x = 5 ∧ y = 4) and y = 4**

(x = 5 ∧ y = 4) ⇒ y = 4

<weakening/strengthening> T

Hence y = 4 is weaker

**x <= 5 V y = 3 and x = 5 ∧ y = 4**

<weakening/strengthening> x = 5 ∧ y = 4 ⇒ x = 5

<weakening/strengthening> ⇒ x <= 5

<weakening/strengthening> ⇒ x <= 5 V y = 3

Hence x <= 5 V y = 3 is weaker

**T and F**

F ⇒ T

Hence T is weaker

**(∀i | 5 <= i <= 10 : b(i+1) < b(i)) and (∀i | 7 <= i <= 10 : b(i+1) < b(i))**

(∀i | 7 <= i <= 10 : b(i+1) < b(i)) ⇒

<weakening/strengthening> (∀i | 7 <= i <= 10 : b(i+1) < b(i)) V (∀i | 5 <= i < 10 : b(i+1) < b(i)) ⇒

<algebra> (∀i | 5 <= i < 10 : b(i+1) < b(i))

Hence (∀i | 5 <= i < 10 : b(i+1) < b(i)) is weaker

**(x <= 1) and (x >= 5)**

(x <= 1) and (x >= 5) can never both be true

so they never imply each other

So none is weaker than the other

**3.3.1.7) Determine:**

**wp( y := x - 1 , y <= 1)** ⇒ x - 1 <= 1 ⇒ x <= 2

**wp( x := x - 1 , x <= 1)** ⇒ x - 1 <= 1 ⇒ x <= 2

**wp( a := a + b(i) , a = ∑ (k=0) i-1 b(k))**  ⇒ a = b(i) + ∑ (k=0) i-1 b(k)

⇒ a = ∑ (k=0) i b(k)

**3.3.2.8) Prove:**

**(p ⇒ r ∧ q ⇒ r) ⇔ (p V q) ⇒ r**

(p ⇒ r ∧ q ⇒ r)

<Implication> (¬p V r) ∧ (¬q V r) ⇔

<distributive> r V (¬p ∧ ¬q) ⇔

<de morgan> r V ¬(p V q)

<implication> (p V q) ⇒ r

**3.3.4.10) Abort satisfies:**

**Excluded Miracles**

wp(abort, R) = F ⇒ wp(abort, F) = F

**Distributivity of Conjunctions**

Let Q and R be arbitrary states

wp(abort, Q) = F

wp(abort, R) = F

Hence wp(abort, Q) ∧ wp(abort, R) = F

wp(abort, Q ∧ R) = F

F = F

F = F ∧ F

Hence wp(abort, Q ∧ R) = wp(abort, Q) ∧ wp(abort, R)

**Monotonicity**

If Q ⇒ R then wp(S,Q) ⇒ wp(S,R)

Let Q and R such that Q ⇒ R

wp(abort, Q) = F

wp(abort, R) = F

F ⇒ F

Hence wp(abort, Q) ⇒ wp(abort, R)

**Distributed Disjunctions**

wp(abort, Q) = F

wp(abort, R) = F

wp(abort, Q V R) = F

F V F ⇒ F

wp(abort, Q) V wp(abort, R) ⇒ wp(abort, Q V R)

**3.3.6.12) Determine WP**

**wp( i := i+1 , i = j )**  = ( i + 1 = j ) = ( i = j - 1 )

**wp( i := i+1 ; j := j+i , i = j )** = wp( i := i+1 , wp(j := j+i , i = j ))

= wp( i := i+1 , i = j + i)

= wp( i := i+1 , j = 0)

= (j = 0)

**wp( i := 2i+1 ; j := j+i , i = j )** = wp( i := i+1 , wp(j := j+i , i = j ))

= wp( i := 2i+1 , i = j + i)

= wp( i := 2i+1 , j = 0)

= (j = 0

**wp( j := j+i ; i := 2i+1 , i = j )** = wp(j := j+i , wp(i := 2i+1 , i = j))

= wp(j := j+i , 2i+1 = j)

= (2i + 1 = j + i)

= (j = i + 1) = (i = j - 1)

**wp(“t := i ; i := j ; j := t”, i = i\_hat ∧ j = j\_hat)**

= wp(“t := i ; i := j” , wp( “j := t” , i = i\_hat ∧ j = j\_hat))

= wp(“t := i” , wp( “i := j” , i = i\_hat ∧ t = j\_hat))

= wp(“t := i” , j = i\_hat ∧ t = j\_hat)

= ( j = i\_hat ∧ i = j\_hat)

**wp(“ i = 0; s := 0”, 0 <= i < n ∧ s = ( Σ j | 0 <= j < i : b(j)))**

= wp( “i = 0” , wp(s := 0”, 0 <= i < n ∧ s = ( Σ j | 0 <= j < i : b(j))))

= wp( i = 0, 0 <= i < n ∧ 0 = ( Σ j | 0 <= j < i : b(j)))

= wp( i = 0 , 0 = ( Σ j | 0 <= j < i : b(j)))

= (0 = ( Σ j | 0 <= j < 0 : b(j)))

= F ( since 0 <= j < 0 is a false condition)

**wp( s := s + b(i) ; i := i + 1, 0 <= i < n ∧ s = ( Σ j | 0 <= j < i : b(j)))**

= wp( s := s + b(i) , wp(i := i + 1, 0 <= i < n ∧ s = ( Σ j | 0 <= j < i : b(j))))

= wp( s := s + b(i) , 0 <= i + 1 < n ∧ s = ( Σ j | 0 <= j < i + 1: b(j)))

= ( 0 <= i + 1 < n ∧ s + b(i) = ( Σ j | 0 <= j < i + 1: b(j))))

= (s = ( Σ j | 0 <= j < i + 1 < n : b(j)))

**3.3.6.13)**

**{ P: 0 <= k < n ∧ a = ( Σ i | 0 <= i <= k : b(i))}**

**S0: k := k + 1**

**S1: a := a + b(k)**

**{ R: 0 <= k <= n ∧ a = ( Σ i | 0 <= i <= k : b(i))}**

Proof:

wp(S0; S1, R) = wp(S0 , wp(S1, R)) =

<assignment> wp( k := k+1, 0 <= k <= n ∧ a + b(k) = ( Σ i | 0 <= i <= k : b(i))) =

<assignment> (0 <= k+1 <= n ∧ a + b(k+1) = ( Σ i | 0 <= i <= k+1 : b(i)))) =

<algebra> (0 <= k < n ∧ a = ( Σ i | 0 <= i <= k : b(i)))) =

P